CLASS24

PAPER CODE	-
FORM NUMBER	

Genius Seed Program

(ACADEMIC SESSION 2023 – 2024)

National Mathematics Talent Contest 2023

MOCK TEST – 1 (PRIMARY)

Time : 3 Hours

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- Rulers and compasses are allowed.
- Answer all questions. Each question carries 10 marks.
- Elegant and innovative solutions will get extra marks.
- Diagrams and justification should be given wherever necessary.
- Before answering, fill in the FACE SLIP completely.
- Your 'rough work' should be in the answer sheet itself.
- The maximum time allowed is THREE hours.

Name of the Candidate (in Capitals) _____

Form Number : _____

Centre of Examination (In Capitals) : _____

Candidates's Signature : Invigilator's Signature :

CLASS24	National Mathematics Talent Contest-2023	NMTC (STAGE– II)

Time : 3 hours **Mathematics : Mock Test -1**

1.

(i) Find the total number of digits in the number 1234 .... 2021 2022 2023.

Solution: Number of digits' is counted as follows:

No. of single digit numbers written are {1, 2, ..., 9 } 9

No. of 2-digit numbers written are {10, 11, ..., 99 } 90

No. of 3 digit numbers written are {100,101, ... , 999} 900

No. of 4 digit numbers written are {1000, 1001, ..., 2022,2023} 1024

Total No. of digits in the given number $= 9\times1 + 90 \times 2 + 900 \times 3 + 1024 \times 4$

$= 9 + 180 + 2700 + 4096$

$= 6985.$

(ii) Find the sum of all the numbers representing the dates of the 12 months from January to December 2024.

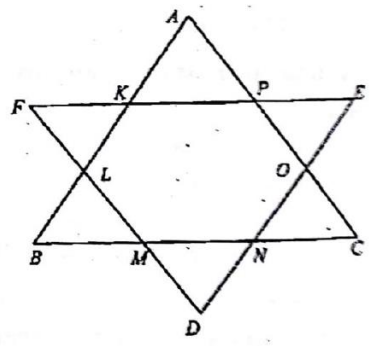
Solution: There are 7 months with 31 days.

4 months with 30 days and 1 month with 29 days.

Sum of all the dates $= (1 + 2 +...+ 31) \times 7 + (1 + 2 +....+ 30)\times4 + (1+2+.... +29)$

$= 3472+1860+435 = 5767$

2. ABC is an equilateral triangle. The pair of points (K,L) (M,N) and (O,P) divide the sides AB,BC and CA respectively into three equal parts. The lines through (K,P),(L,M) and (N,O) enclose the triangle DEF as shown in the figure. Prove that the triangle DEFis equilateral.



Solution: ABC is Equilateral triangle.

$AK = KL = LB = BM = MN = NC = CO = OP = AP$

In $\triangle AKP$

$$\angle A = 60^\circ$$

and $AK = AP$

$$\therefore \angle AKP = \angle APK = \frac{180 - 60}{2} = 60^\circ$$

Similarly $\triangle BLM$ and CNO are equilateral triangles.

$$\angle BLM = \angle BML = 60^\circ = \angle CNO = \angle CON$$

In $\triangle FKL, \angle FKL = \angle AKP$ (vertically opp angle)

$= 60^\circ$

$\angle FLK = \angle BLM = 60^\circ$

(Vertically opposite angles)

$\angle KFL = 60^\circ$

Similarly in $\triangle DMN$

$\angle MDN = 180^\circ - \angle DMN - \angle DNM$

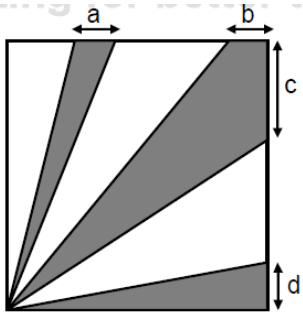
$= 180^\circ - \angle BML - \angle CNO$

$= 180^\circ - 60^\circ - 60^\circ = 60^\circ$

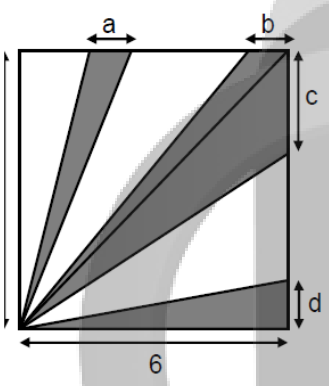
$\angle OEP = 60^\circ$

$\therefore \triangle DEF$ has all its angles equal to 60°. $\therefore$ $\triangle DEF$ is equilateral.

3. Inside a square of area 36 cm^2, there are shaded regions as shown. The ratio of the shaded area to the unshaded area is 3 : 1. What is the value of a + b + c + d where a, b, c, d are the lengths of the bases of the shaded regions ? Further, if three of a, b, c, d are equal integers and one different, then find them.



Sol.



Shaded area = $\frac{1}{2} \times 6 \times a + \frac{1}{2} \times 6 \times b + \frac{1}{2} \times 6 \times c + \frac{1}{2} \times 6 \times d$

$3(a + b + c + d)$

So unshaded area = $36 - 3(a + b + c + d)$

A.T.Q.

$$\frac{3(a+b+c+d)}{36-3(a+b+c+d)} = \frac{3}{1}$$

$(a + b + c + d) = 36 - 3(a + b + c + d)$

$4(a + b + c + d) = 36.$

$\Rightarrow \quad a + b + c + d = 9$

If $a = b = c = 1$ then $d = 6$ (not possible)

If $a = b = c = 2$ then $d = 3$ (possible)

so equal three integers are 2 and different is 3.

4.

A, M, T, I represent different non-zero digits, It is given that

A + M + T + I – 11

A + M + I – 10

A + M – I

Further in the following addition only one digit is given.

A M T I A M T I

A M T I A M T

A M T I A M

A M T I A

A M T I

A M T

A M

A

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* * * * 5 * * *

Fill up the stars writing proper reasons.

Sol.

A + M + T + I = 11 ..(i)

A + M + I = 10 ..(ii)

A + M = I ..(iii)

from equation (i) and (ii) T = 1

from equation (2) and (3) I = 5

A + M = 5

The first row given addition as = 22

The next row will come by adding [T + M + A + I + T + M + A + 2] = 19

The third row = [I + M + A + I + T + M + A] = 17

The 4th row = [I + A + I + T + M + A] = 13 + A = 15 and A = 2 and M = 3

∴ The sum is 35725792

5. There are 4 girls and 2 boys of different ages. The eldest is 10 years old while the youngest is 4 years old. The older of the boys is 4 years older than the youngest of the girls. The oldest of the girls is 4 years older than the youngest of the boys. What is the age of the oldest of the boys?

Sol.

Solution: Let the ages of the girls be g_1, g_2 , g_3, g_4 and the ages of the boys b_1 ,b_2. We can assume that $g_1 < g_2 < g_3 < g_4$ and $b_1 < b_2$. If the eldest is a boy, then $b_2 = 10$ and $g_1 = 10 - 4 = 6$. The youngest must be a boy and thus $b_1 = 4$. The oldest of the girls is 4 years older than the youngest of the boys. Hence $g_4 = 8$. But then $g_1 = 6$, $g_4 = 8$ and gi < g2 < #3 < P4 is not possible since there is only one number between 6 and 8. Consequently, the eldest must be a girl and $g_4 = 10$. This implies that $b_1 = 6$ and thus the youngest must be a girl aged 4 years. Thus the age of the older boy is 8 years.

6.(i) Two numbers are respectively 20% and 50% more than a third number. What percentage is the first of the second?

ii) Three vessels of sizes 3 litres, 4 litres and 5 liters contain mixture of water and milk in the ratio 2:3, 3:7 and 4:11 respectively. The contents of all the three vessels are poured into a single vessel. What is the ratio of water to milk in the resultant mixture?

Sol.

(a) Let a, b be the two numbers and the third number be X. Given that $a = X + \frac{20}{100}X = \frac{6}{5}X$ and $b = X + \frac{50}{100}X = \frac{3}{2}X$. Dividing the above two equations, we get $\frac{a}{b} = \frac{4}{5}$. So the first is 80% of the second.

(b) The actual volume of water and milk in the 3 vessels are:

	Water	Milk
1st Vessel	$\frac{6}{5}$	$\frac{9}{5}$
2nd Vessel	$\frac{12}{10}$	$\frac{28}{10}$
3rd Vessel	$\frac{20}{15}$	$\frac{55}{15}$

Thus the ratio of the water to milk when the contents of the vessels are poured into a single vessel is

$$\frac{\frac{6}{5} + \frac{12}{10} + \frac{20}{15}}{\frac{9}{5} + \frac{28}{10} + \frac{55}{15}} = \frac{\frac{56}{15}}{\frac{124}{15}} = \frac{56}{124} = \frac{14}{31}$$

The ratio of water to milk in the resultant mixture is 14 : 31.